

Phase transition in a triplet process

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We argue that the reaction-diffusion process $3A \rightarrow 4A, 3A \rightarrow 2A$ exhibits a different type of continuous phase transition from an active into an absorbing phase. Because of the upper critical dimension $d_c \geq 4/3$ we expect the phase transition in 1+1 dimensions to be characterized by nontrivial fluctuation effects.

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The classification of continuous phase transitions far from thermal equilibrium is one of the most challenging tasks of modern statistical physics [1]. Within this field many studies are concerned with phase transitions from a fluctuating active phase into one or several nonfluctuating absorbing states, which are believed to be associated with a finite number of universality classes [2]. For such a phase transition to occur it is necessary that (a) at least one absorbing state is dynamically accessible, (b) there are two competing processes for particle creation and removal, and (c) there is a mechanism which prevents the particle density from diverging.

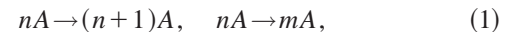
In many cases, the critical behavior close to the transition is characterized by simple power law scaling. In sufficiently high dimensions the critical exponents are given by their mean-field values, whereas below a certain upper critical dimension d_c fluctuation corrections have to be taken into account, leading to nontrivial exponents and scaling functions. For this reason the study of fluctuation effects in low dimensional, especially (1+1)-dimensional systems is particularly interesting.

The most important universality class of absorbing phase transitions is directed percolation (DP), which occurs in all processes following the reaction-diffusion scheme $A \leftrightarrow 2A, A \rightarrow \emptyset$. The critical exponents, especially in one spatial dimension, are known to a very high precision [3]. The critical behavior of DP can be described in terms of a renormalizable field theory which was originally introduced in the context of high energy physics [4]. DP is relevant for experimental applications such as catalytic reactions [5], flowing sand [6], and spatiotemporal intermittency of magnetic fluids [7].

The other established class is the parity-conserving (PC) universality class. This type of critical behavior is observed in a large variety of models which can be divided into two groups. The first group includes all parity-conserving particle processes [8–11] such as branching-annihilating random walks with two offspring $A \rightarrow 3A, 2A \rightarrow \emptyset$. The second group of models comprises spreading processes with two symmetric absorbing states, including kinetic Ising models [12], interacting monomer-dimer models [13], as well as generalized versions of the Domany-Kinzel model and the contact process [14]. In higher dimensions the second group of models describes branching-annihilating *interfaces* and can be asso-

ciated with the voter universality class [15]. Only in 1+1 dimensions the two classes of models exhibit the same type of critical behavior.

Recently the pair contact process with diffusion (PCPD), also called annihilation-fission process, attracted considerable attention. The PCPD is a *binary* spreading process following the reaction-diffusion scheme



with $n=2, m \leq 1$. It exhibits a continuous phase transition and thus could serve as a candidate for another independent universality class. The PCPD was already suggested in 1982 by Grassberger [16], but it took almost 20 years until Howard and Täuber presented a first systematic study of a bosonic variant of the process [17]. Using field-theoretic methods they were able to prove the existence of a phase transition, although the corresponding field theory turned out to be unrenormalizable. More recently, several authors studied various fermionic variants of the PCPD [18–25]. Meanwhile there is a general consensus that the critical behavior of the PCPD is different from all other previously known universality classes. However, it turned out to be extremely difficult to estimate the critical exponents in a reliable way, mainly because of unusually strong deviations from ordinary power-law scaling.

In the present study we investigate the question whether further different types of critical behavior will emerge for $n > 2$. In particular we will focus on the case $n=3$, called triplet process. As will be shown below, we argue that this process exhibits yet another different type of critical behavior.

(a) *Mean field approximation.* In order to determine critical dimension d_c and the mean field critical exponents of the reaction-diffusion process (1) for general $m < n$, let us consider a simple mean field theory. We expect this process to be described by the Langevin equation

$$\partial_t \rho(\mathbf{x}, t) = a \rho^n(\mathbf{x}, t) - \rho^{n+1}(\mathbf{x}, t) + D \nabla^2 \rho(\mathbf{x}, t) + \zeta(\mathbf{x}, t), \quad (2)$$

which for $n=1$ reduces to the well-known Langevin equation for DP [26]. The first term accounts for both particle creation and removal so that the parameter a plays the role of the reduced spreading probability $p - p_c$. The second term is

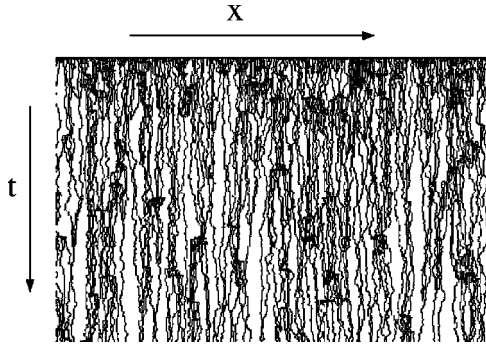


FIG. 1. One-dimensional triplet process at criticality starting with a fully occupied initial state.

the most relevant contribution preventing the particle density from going to infinity, while the third term describes nearest-neighbor diffusion.

The noise $\zeta(\mathbf{x}, t)$ takes the stochastic nature of particle creation and removal into account. Its amplitude has to depend on the local density $\rho(\mathbf{x}, t)$ since in the absorbing state $\rho=0$ there are no density fluctuations. Thus it is near at hand to expect noise correlations of the form

$$\langle \zeta(\mathbf{x}, t) \zeta(\mathbf{x}', t') \rangle = \Gamma \rho^\mu(\mathbf{x}, t) \delta^d(\mathbf{x} - \mathbf{x}') \delta(t - t') \quad (3)$$

with an unknown exponent μ . For DP ($n=1$), where the squared noise amplitude is proportional to the density of particles, this exponent is given by $\mu=1$. For $n>1$, however, the situation is more involved. Without the branching process, i.e., deep in the inactive phase, the squared noise amplitude is proportional to $\partial_t \rho(\mathbf{x}, t)$, hence $\mu=n$. At the transition, however, the branching process may lead to positive correlations among the particles, increasing the intensity of the noise and thereby reducing the value of μ . At criticality we therefore expect μ to be in the range

$$1 \leq \mu \leq n. \quad (4)$$

Solving the stationary mean field equation $0 = a\rho^n - \rho^{n+1}$ we obtain the stationary density $\rho=a$, hence the critical point is $a_c=0$ and the density exponent is $\beta^{\text{MF}}=1$. At the mean field critical point the full Langevin equation should be invariant under the scaling transformation

$$\mathbf{x} \rightarrow \Lambda \mathbf{x}, \quad t \rightarrow \Lambda^z t, \quad \rho \rightarrow \Lambda^{-\chi} \rho, \quad (5)$$

where Λ is a dilatation factor while $z = v_{\parallel}/v_{\perp}$ and $\chi = \beta/v_{\perp}$ are quotients of the three standard critical exponents. Comparing all terms except for the noise, scaling invariance implies that $z=2$ and $\chi=2/n$, i.e.,

$$\beta^{\text{MF}}=1, \quad v_{\perp}^{\text{MF}}=n/2, \quad v_{\parallel}^{\text{MF}}=n. \quad (6)$$

Moreover, we can check the relevance of the noise term, which is responsible for fluctuation effects. By simple power counting we find that the noise is relevant below the upper critical dimension

$$d_c = 2 + \frac{4-2\mu}{n}, \quad (7)$$

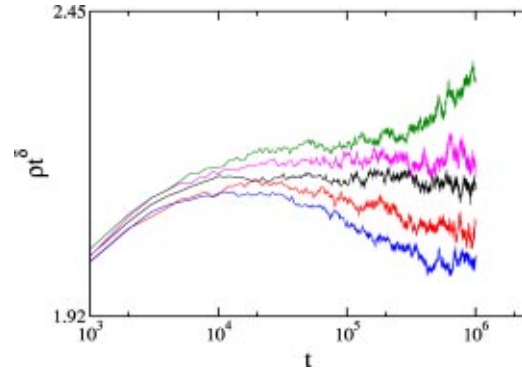
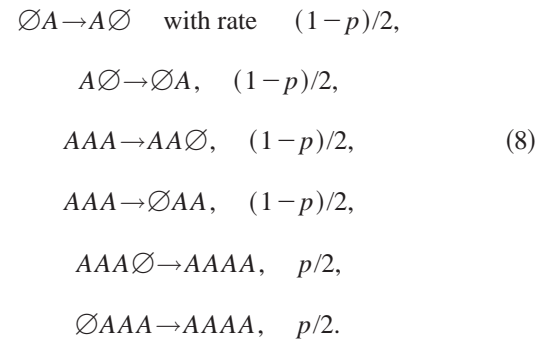


FIG. 2. The density of particles $\rho(t)$ times t^δ ($\delta=0.32$) as a function of time for $p=0.6855, 0.6853, 0.6851, 0.6849,$ and 0.6847 from top to bottom, averaged over 1500 runs on a system with 4096 sites. The best straight line is obtained for $\delta=0.32$ and $p=0.6851$.

while it is irrelevant above d_c where the mean field exponents (6) are expected to become exact. For DP ($n=\mu=1$) we obtain the well-known result $d_c=4$, while for the PCPD the upper critical dimension has to be in the range $2 \leq d_c \leq 3$. This result is in agreement with recent numerical findings by Ódor *et al.* [25] suggesting that $d_c=2$.

As the main observation, which triggered the present work, we note that the upper critical dimension for third-order processes ($n=3, m<n, 1 \leq \mu \leq 3$) is larger than $4/3$. Consequently, in $1+1$ dimensions such a triplet process (TP) should still be characterized by nontrivial fluctuation effects. Moreover, the density in the inactive phase is known to decay as $\rho(t) \sim (\ln t/t)^{1/2}$. This type of decay in the absorbing phase differs from all other previously known universality classes of phase transitions into absorbing states, suggesting that also the transition itself should belong to yet another universality class.

(b) *Numerical simulations.* In order to study the triplet process numerically, we introduce a fermionic variant of the reaction-diffusion process (1) with $n=3$ and $m=2$. It evolves by random-sequential updates and is defined by the following dynamic rules:



A typical space-time plot of the process at criticality is shown in Fig. 1. As can be seen the process generates spatiotemporal structures, possibly indicating the presence of fluctuation effects. Note that we tuned the rates for diffusion and particle removal in the same way as in Ref. [23].

Performing standard Monte Carlo simulations (see, e.g., Ref. [2]) we find clear evidence for a continuous phase tran-

sition between an active phase, where the density of particles is asymptotically constant, and an inactive phase, where the particle density decays algebraically with logarithmic corrections. Assuming that the critical behavior at the transition obeys simple power law scaling we find the critical threshold $p_c = 0.6851(4)$ (see Fig. 2). As in the PCPD, there are strong corrections so that the scaling regime, if existent at all, is not reached before 10^4 time steps. Averaging over many independent runs in the time interval $10^4 < t < 10^6$ we estimate the critical exponents by

$$\nu_{\parallel} = 2.5(2), \quad z = 1.75(10), \quad \delta = \beta/\nu_{\parallel} = 0.32(1). \quad (9)$$

Similar exponents were obtained in other variants of the triplet process with $m < n = 3$ (not reported here). In all cases the estimates differ from the mean field (MF) exponents $\nu_{\parallel}^{\text{MF}} = 3$, $z^{\text{MF}} = 2$, and $\delta^{\text{MF}} = 1/3$, leading us to the conclusion that critical behavior of the 1+1-dimensional TP is indeed characterized by nontrivial fluctuation effects. As expected, these deviations are quite small (less than 20%) since the simulations are carried out close to the upper critical dimension.

We note that our results are not accurate enough to doubtlessly confirm the validity of power-law scaling over a large range. As our simulations seem to reach the scaling regime

only after 10^4 time steps, the accuracy of our estimate for p_c is limited as well. In addition, the asymptotic power-law behavior may be shadowed by logarithmic corrections which are also present in the inactive phase. Finally, as in the case of the PCPD, the assumption of simple power-law scaling and the concept of universality may be questioned as a whole. Nevertheless, we believe that the MF arguments and the numerical evidence are strong enough to conclude that this model exhibits a different type of critical behavior, where fluctuation effects are likely to play an important role.

Regarding the limited accuracy of numerical simulations a major drawback could be the definition of the model as a fermionic reaction-diffusion process with four-site interactions. A bosonic variant with two-site interactions is currently under investigation. Moreover, it is important to determine the exponent μ in the noise correlator. Preliminary simulations suggest a value close to $\mu \approx 2$. Finally, the influence of the diffusion rate has to be studied systematically.

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